Nonlinear Coupling of Kinetic Alfven Waves (KAWs) and Ion Acoustic Waves (IAWs)

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Abstract

The Alfven wave dispersion relation modifies into Kinetic Alfven wave KAW when the perpendicular wavelength becomes comparable to the ion gyro radius. The inclusion of ion motion produces nonlinear coupling of KAW with ion acoustic wave through the coupling parameter λ_s. These coupled KAW are believed to play dynamic role in plasma heating, particle acceleration and anomalous transport. Here we have investigated the properties of this coupled Solitary KAWs by using two potential theory and Sagdeev potential approach. We employ generalized (r, q) distribution function and the numerical results are promising to give excellent fits with observations of Freja, FAST and MMS. We have shown that there are regions of propagation and non-propagation for such solitary structure which is largely dependent upon angle of propagation.

Governing set of Equations

Low β assumption, oblique propagation allow us to use two potential theory

$$\frac{m_e}{m_i} < \beta < 1$$

$$\boldsymbol{E}_x = -\frac{\partial \phi}{\partial x}, \boldsymbol{E}_z = -\frac{\partial \psi}{\partial z}, \boldsymbol{E}_{y=0}$$

The closed set of equations is

$$m_i\left(\frac{\partial \boldsymbol{v_i}}{\partial t} + (\boldsymbol{v_i}.\boldsymbol{\nabla})\boldsymbol{v_i}\right) = e(\boldsymbol{E} + \boldsymbol{v_i} \times \boldsymbol{B}_o)$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_{ix}) + \frac{\partial}{\partial z}(n_i v_{iz}) = 0$$

$$\frac{\partial j_z}{\partial z} = e\frac{\partial n_e}{\partial t} + e\frac{\partial}{\partial z}(n_i\, v_{iz})$$

$$\frac{\partial^4}{\partial x^2 \partial z^2}(\phi - \psi) = \mu_o \frac{\partial^2}{\partial t\, \partial z} j_z$$

To find density, we use distribution function as next

Model Distribution

$$f_{rq}(v) = \frac{A}{\pi\, B^{3/2}\, (v_{th})^{3/2}}\left[1 + \frac{1}{q-1}\left(\frac{v^2 - 2e\psi/m_e}{c\,(2T_e/m_e)}\right)^{r+1}\right]^{-q}$$

$$A = \frac{3\,\Gamma[q]\,(q-1)^{-3/(2+2r)}}{4\,\Gamma\left[q - \frac{3}{2+2r}\right]\Gamma\left[1 + \frac{3}{2+2r}\right]} \qquad B = \frac{3\,(q-1)^{-1/(1+r)}\,\Gamma\left[q - \frac{3}{2+2r}\right]\Gamma\left[\frac{3}{2+2r}\right]}{2\,\Gamma\left[q - \frac{5}{2+2r}\right]\Gamma\left[\frac{5}{2+2r}\right]}$$

$$n_e = n_{eo}[1 + \alpha_1\,\Psi + \alpha_2\,\Psi^2]$$

$$\alpha_1 = \frac{(q-1)^{-\frac{1}{(1+r)}}}{2\,C}\frac{\Gamma\left(\frac{1}{2(1+r)}\right)\,\Gamma\left(q - \frac{1}{2(1+r)}\right)}{\Gamma\left(\frac{3}{2(1+r)}\right)\,\Gamma\left(q - \frac{3}{2(1+r)}\right)}$$

$$\alpha_2 = \frac{-(q-1)^{-\frac{2}{(1+r)}}(1+4r)}{8\,C^2}\frac{\Gamma\left(-\frac{1}{2(1+r)}\right)\,\Gamma\left(q + \frac{1}{2(1+r)}\right)}{\Gamma\left(\frac{3}{2(1+r)}\right)\,\Gamma\left(q - \frac{3}{2(1+r)}\right)}$$

In the limit $r \to 0$ & $q \to \infty$

$$\alpha_1 \to 1, \quad \alpha_2 \to \frac{1}{2}$$

$$n_e = n_{eo}\left(1 + \Psi + \frac{1}{2}\Psi^2\right)$$

Linear Analysis $(\omega << \Omega_i)$

$$\left(1 - \frac{V_A^2 {k_z}^2}{\omega^2}\right)\left(1 - \frac{{c_s}^2 {k_z}^2}{\alpha_1 \omega^2}\right) = \frac{V_A^2 {k_z}^2}{\alpha_1 \omega^2}\lambda_s$$

$$\lambda_s = k_x^2 \rho_s^2$$

$$V_A = \frac{B_o}{\mu_o n_i\, m_i}$$

Nonlinear Analysis

Normalizations

We get stationary solution using comoving frme $\xi = K_x X + K_z Z - M\tau$,
Also normalize variables as $N = \frac{n_{e,i}}{n_o}$, $\Phi = \frac{e\,\phi}{T_e}$, $\Psi = \frac{e\,\psi}{T_e}$, $M = \frac{v_p}{c_s}$, $T = \Omega_i\, t$, $V = \frac{v}{c_s}$, $l = \frac{l}{\rho_i}$

we obtain the following equation which is analogous to the energy integral. Where S(Ψ) is the Pseudopotential or so-called Sagdeev potential.

$$\frac{1}{2}\left(\frac{\partial \Psi}{\partial \xi}\right)^2 + S(\Psi) = 0$$

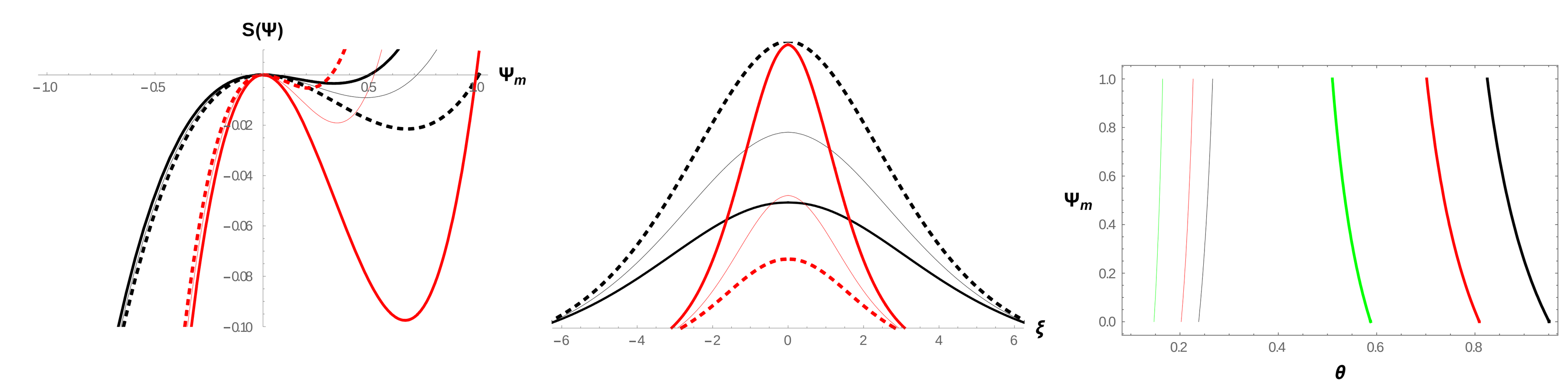
$$S(\Psi) = \frac{1}{K_x^2}\left[\left(-\alpha_1\left(1 - \frac{M_A^2}{{K_z}^2}\right) - \frac{\beta}{2}\left(1 - \frac{{K_z}^2}{M_A^2}\right)\right)\frac{\Psi^2}{2} + \left({\alpha_1}^2\frac{{M_A}^2}{{K_z}^2} - \beta\alpha_1\left(\frac{3}{2} - \frac{{K_z}^2}{{M_A}^2}\right) - \alpha_2\left(1 - \frac{{M_A}^2}{{K_z}^2}\right)\right)\frac{\Psi^3}{3}\right]$$

Numerical Analysis

We can get compressive solitary structures for following conditions of Sagdeev Potential, we get two regions of propagation.

$$S(\Psi)|_{\Psi=0} = 0 \qquad S'(\Psi)|_{\Psi=0} = 0 \qquad S''(\Psi)|_{\Psi=0} < 0$$

$$S(\Psi)|_{\Psi=\Psi_{max}} = 0 \qquad S'(\Psi)|_{\Psi=\Psi_{max}} > 0$$



Sagdeev potentials (Left), corresponding compressive solitons (center) for different existence regions and Mach numbers M_A=0.257 (bold), 0.258 (thin), 0.259 (dashed), M_A=0.85 (bold), 0.88 (thin), 0.9 (dashed) when r= 2, $q = 3$, $\theta = 0.8\,\frac{\pi}{2}$ and $\beta = 0.1$ and maximum value of potential (right) for different propagation angles.

Results

1. For lower range of Alfven Mach number, width and amplitude of soliton increases but it shows opposite behavior for region of lower Alfven Mach number. The value of potential is much higher for higher velocity region as compared to low velocity region.
2. Moreover, since the parameter ξ is normalized by the ion-acoustic larmor radius, it shows that the maximum scale length of the formation of solitary structures varies from one sixth of a Kilometer for higher velocities to one twelfth of a Kilometer for lower range of velocities.
3. For low-velocity region, maximum potential increases with increase in angle of propagation. In contrast to that, for high-velocity range maximum potential increases with decrease in propagation angle.

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